Math 18A Support Notes
Topic: Factoring, Rationalizing, Clearing Fractions, and Trigonometric Identities
Support for: Evaluating Limits Algebraically

Key Concepts: Factoring trinomials, quadratic form, factor by grouping, factoring differences of squares $a^2 - b^2 = (a-b)(a+b)$, factoring sum and difference of cubes $a^3 \pm b^3 = (a \pm b)(a^2 \mp +ab + b^2)$, multiplying by a conjugate, simplifying complex fractions, double angle identity $\sin 2x = 2\sin x \cos x$, Pythagorean identity $\sin^2 x + \cos^2 x = 1$

1. Factor completely.
   a) $6x^2 - 23x - 4$
   b) $21x^2 + x - 10$
   c) $x^4 + 2x^2 - 24$
   d) $8x^3 - 4x^2 - 2x + 1$
   e) $e^{2x} - 2e^x + 1$
   f) $e^{3x} - e^{5x}$
   g) $\sin 2x - \sin x$
   h) $8x^3 + 27$

2. Rewrite the expression by multiplying by the conjugate of the numerator: $\frac{\sqrt{16+h}-4}{h}$

3. Simplify a) $\frac{\frac{4}{(3+h)} - \frac{4}{3}}{h}$ b) $\frac{\cos^2 x}{\sin x - 1}$

Rate your overall understanding of these topics: A B C D F
Notes to self: ____

Math 180 Calculus 1 Notes

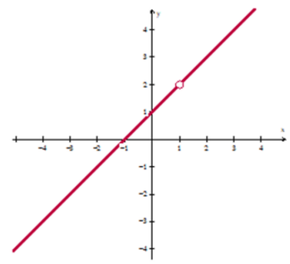
Limits (Part 4)

Evaluating Limits Algebraically and More (Stewart 2.3)

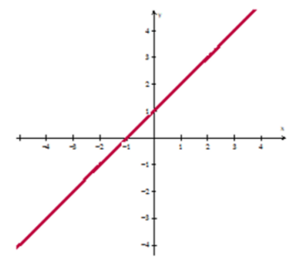
(Review) **Theorem**: If a function is continuous at $x = c$, then the limit is equal to the function value obtained by direct substitution, so $\lim_{x \to c} f(x) = f(c)$.

What we do if the function is discontinuous at $x = c$?

Ex 1: Evaluate $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$



$f(x) = \frac{x^2 - 1}{x - 1}$



$g(x) = x + 1$

Theorem: Let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c. If $\lim_{x \to c} g(x)$ exists, then $\lim_{x \to c} f(x)$ exists and $\lim_{x \to c} f(x) = \lim_{x \to c} g(x)$.

Indeterminate*:* We say that $\lim_{x \to c} f(x)$ is indeterminate if, when attempting to evaluate the limit of $f(x)$ as x approaches c using direct substitution, we obtain an expression of the form*

$$\frac{0}{0} \text{ or } \infty - \infty.$$

*Note: There are other indeterminate forms we will discuss later in the semester.

Strategies for Algebraic Manipulation of Limits*:* If direct substitution results in an indeterminate form we can use the following algebraic strategies to rewrite the function and then use substitution because the original function and the rewritten function agree at all but one point.

1. Factor (or expand) and cancel
2. Multiply by the conjugate and simplify
3. Combine fractions (or clear fractions) and simplify

Ex 2*:* Find $\lim_{x \to 0^+} \frac{e^{3x} - e^x}{e^x - 1}$

Ex 3: Find $\lim_{t\to0}\frac{\sqrt{t+1}-1}{t}$

Direct substitution yields $\frac{\sqrt{0+1}-1}{0}=\frac{\sqrt{1}-1}{0}=\frac{0}{0}$ indeterminate

$$\lim_{t\to0}\frac{\sqrt{t+1}-1}{t}$$

Ex 4: Find $\lim_{h\to0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

Direct substitution yields $\frac{\frac{1}{(x+0)^2}-\frac{1}{x^2}}{0}=\frac{0}{0}$ indeterminate

$$\lim_{h\to0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

Ex 5: Evaluate $\lim_{y \to 0} \left(\frac{2}{y} - \frac{2}{y^2+y}\right) =$

The next two examples use some different strategies to determine the limit.

Ex 6: Find $\lim_{x \to 0} \frac{\sec x - 1}{\tan^2 x}$

Direct substitution yields $\frac{\sec 0 - 1}{\tan^2 0} = \frac{1-1}{0} = \frac{0}{0}$ indeterminate

$$\lim_{x \to 0} \frac{\sec x - 1}{\tan^2 x}$$

Ex 7: Evaluate $\lim_{x \to 5} \frac{|x-5|}{x-5} =$

Recap: If direct substitution results in an indeterminate form like $\frac{0}{0}$, then $\frac{0h}{n0}$ you need to do more work to see if you can use other techniques to make it determinate! The answer is <u>NOT</u> always does not exist!

Name: ______________________________________

Math 180 Check Your Understanding: Algebraic Limits Video Lesson

Pre-Class Reflection: After watching the video lesson, how well do you feel you understand the material presented so far? (Circle one)

Not at all   Somewhat well   Very well   Extremely well

Use your notes to help you complete the following problems. If you are struggling with all the problems below, be sure to have specific questions ready. You are also advised to rewatch the video or search YouTube for another video on the topic.

Evaluate the following limits.

1. $\lim_{x \to 3} \frac{x-3}{x^2-9}$
2. $\lim_{x \to 1} \frac{x-1}{1-\sqrt{x}}$
3. $\lim_{x \to 0} \frac{e^x-1}{e^{2x}-e^x}$

What questions do you have so far on this lesson, if any?

Name: ____

Math 180 Assignment: Limits (Part 4)

Algebraic Limits

Required Reflection [must be done in a different color]:

After trying every problem, review the answers and put a "✔" to the left of the problem if you were correct. If incorrect, put an "**X**" and **write the full correct solution.** Circle the problem number for any you are still struggling to understand.

- How many problems are you still struggling to understand? __________ (If none, write none)
- Rate your current level of understanding on this topic: A B C D F

Evaluate each limit using the appropriate method. Show all work and be sure to correctly use the limit notation.

1. $\lim_{x \to 5} \frac{x^2-6x+5}{x-5}$

2. $\lim_{t \to -3} \frac{t^2-9}{2t^2+7t+3}$

3. $\lim_{h \to 0} \frac{(-5+h)^2-25}{h}$

4. $\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$

5. $\lim_{x \to 5} \frac{x^2-25}{x^2-5x-6}$

6. $\lim_{x \to 0} \frac{\cos^2 x-1}{1-\cos x}$

7. $\lim_{x \to -2^+} \frac{x+2}{x^3+8}$ (Hint: Sum of cubes formula)

8. $\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2}$

9. $\lim_{h \to 0} \frac{(x+h)^3-x^3}{h}$

10. $\lim_{x \to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

11. $\lim_{t \to 2} \frac{t-2}{|2-t|}$

12. $\lim_{x \to 3} \left(\frac{2}{x-3} - \frac{12}{x^2-9}\right)$

Spiral Review

13. Evaluate $\lim_{x \to -3} \frac{x^2+9}{(x+3)^2}$

14. Evaluate $\lim_{x \to 5^+} \frac{x^2-4x-5}{x^2-10x+25}$

15. Evaluate $\lim_{x \to 0} (\sin^2 x + \cos^2 x)$

16. Evaluate $\lim_{x \to 1} \arctan x$